

Final Exam
DMATH-III: MATH 3
Fall 2025

Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration: 2 hours
Instructor's Name: Dr. Rokhsaneh Yousef Zehi & Muhammad Javed
Exam Date: 24/12/2025
Program: DO

| | |
|--|-----------|
| | 40 |
| | 10 |

Student Information

Name: ID:
Signature:

Invigilator

Initials: Student ID checked
Time received:

Question 1**[4 marks]**

A cube has side length x cm, and the side length (x) is increasing at a rate of $0.5 \text{ cm}/\text{min}$.

(a) Determine the rate of change of the volume with respect to time when $x = 6$ cm.

(2 marks)

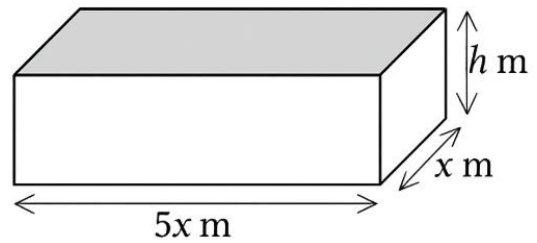
(b) Determine the rate of change of the surface area with respect to time when $x = 6$ cm.

(2 marks)

Question 2**[5 marks]**

A rectangular tank is to be made in the shape of a cuboid. The container has a length of $5x$ m, a width of x m, and a height of h m. The total external surface area of the container must be 60 m^2 . The aim is to construct it with the maximum possible volume.

Determine the value of x that will maximize the tank's volume.



Question 3**[6 marks]**

Find all the stationary points of the following function and then determine whether each one is a maximum, minimum, or point of inflection.

$$f(x) = x^4 - 8x^2$$

Question 4**[5 marks]**

Calculate the following integrals:

(a) $\int (3x^2 + 6x + 4x^{-2} - 5) dx$

(2 marks)

(b) $\int \left(\frac{2}{\sqrt{t}} - 3t^3 + t^3\sqrt{t} \right) dt$

(3 marks)

Question 5**[5 marks]**

Approximate the value of the definite integral

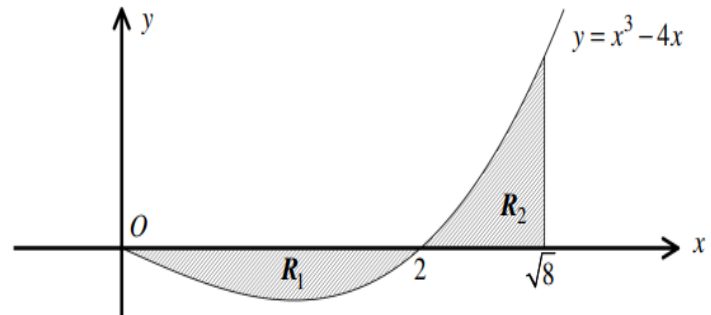
$$\int_2^5 \frac{1}{x^2 + 1} dx$$

using the **Simpson's Rule** with $n = 6$ subintervals. Write your answer correct to 3 decimal places.

Question 6**[5 marks]**

The area between the curve $y = x^3 - 4x$ and the x -axis from $x = 0$ to $x = \sqrt{8}$ is shown shaded in two parts, R_1 and R_2 , in the diagram below.

Determine the total shaded area $R_1 + R_2$.



Question 7**[5 marks]**

A ship's freshwater tank is created by revolving the curve

$$y = \frac{1}{2}\sqrt{36 - x^2}$$

around the **y-axis**, where y is the height (in m) and x is the horizontal distance (in m).

Using **Disk method**, determine the volume of the tank, formed by revolving this curve from $y = 0$ to $y = 3$.

Question 8**[5 marks]**

The time rate of change of displacement (velocity) of a small boat travelling along a river is given by:

$$\frac{ds}{dt} = \frac{50t}{(t^2 + 1)^3}$$

where s is the displacement (in km) and t is the time (in hours).

Determine the total displacement (s) of the boat between $t = 1$ hour and $t = 3$ hours.

Formula sheet:

| | |
|--------------------------|--|
| Volume of a Cube | $V = x^3$ |
| Surface area of a Cube | $A = 6x^2$ |
| Surface area of a Cuboid | $A = 2(lw + lh + wh)$ |
| Volume of a Cuboid | $V = l \times w \times h$ |
| Simpson's rule | $\int_a^b f(x)dx \approx \frac{h}{3} [y_0 + 4y_1 + 2y_2 + \dots + 4y_{n-1} + y_n]$ |

MLO and Bloom's Level of Complexity

| Q # | MLO Addressed | Complexity Level | Mark | Remark |
|-----|---------------|-----------------------|------|--------|
| 1 | 2,3 | Application, Analysis | 4 | |
| 2 | 3,4 | Analysis, Evaluate | 5 | |
| 3 | 1,2 | Application | 6 | |
| 4 | 1 | Application | 5 | |
| 5 | 1 | Application | 5 | |
| 6 | 1 | Application | 5 | |
| 7 | 2,3 | Application, Analysis | 5 | |
| 8 | 2,3 | Application, Analysis | 5 | |

References:

1. J. Washington, A., 2014. Basic Technical Mathematics with Calculus. 10 ed. Harlow: Pearson Education Limited.
2. Stewart, J., 2008. *Calculus: Early Transcendentals*. 6th ed. Boston: Brooks/Cole.