

ACADEMIC YEAR 2023 - 2024

Program	Year	Semester	Paper
DO	2	2	MAIN
MODULE NAME:	MATH 3		
MODULE CODE:	DMATH-III	EXAM DATE:	21.05.2024
INSTRUCTOR's NAME:	Dr. Taofeek	DURATION:	2 hrs.

Questions to be answered on:



Space provided on the question paper

Allowed tools:

Pen, Pencil & Calculator

Number of pages

(Incl. cover page): **12**

Points of attention:

- For each question, the maximum earned points are mentioned between brackets at the end of each question.
- Write very clearly! Answers that are not readable are not marked and don't get points!
- Make sure your answers are written to the point.
- All answers should be written **in English**.
- Write all the answers in **blue or black pen only**.
- Use the **pencil** only for **diagrams & graphs**.
- Show all the calculation steps in the given space.
- When finished submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

STUDENT NAME:

STUDENT ID:

FINAL MARKS

	40
	10

Number of answer scripts:.....

Invigilator:.....

Student's signature:

Time of receipt:.....

ANSWER ALL THE QUESTIONS

Question 1

[5 Marks]

If the function $f(x) = 3x + \frac{4}{x}$, then

a) Find the derivative of $f(x)$ using the definition.

(3 marks)

- b) Find the slope of a line tangent to the curve $f(x)$ at the point $(2, 3)$. (2 marks)

Question 2

[4 Marks]

- a. A bullet is fired vertically upward. Its distance s (*in ft*) above the ground is given by (2 marks)

$$s = 2250t - 16t^2$$

where t is the time (*in secs*). Determine the acceleration of the bullet.

- b. A load (in N) is distributed along a beam 10 m long such that $L = 5x - 0.5x^2$, where x is the distance from one end of the beam. Find the expression for the instantaneous rate of change of L with respect to x ? (2 marks)

Question 3

[4 marks]

Evaluate the derivative of $y = \frac{3x^2+x}{1-4x}$ at $(2, -2)$

Question 4

[4 Marks]

Evaluate the second derivative of the given function for the given value of x .

$$f(x) = 8(1 + 2x)^3, x = \frac{1}{4}$$

Question 5

[5 Marks]

For the implicit function

$$3x^2 + xy + 2y^2 + 6 = 0$$

- a) Find the slope of a line tangent to the curve of the function at the point (1, 2).

(3 marks)

- b) Find the equation of the normal line to the function at the point (1, 2).

(2 marks)

Question 6

[5 marks]

A small oil refinery estimates that its daily profit P (in dollars) from refining x barrels of oil is

$$P = 8x - 0.02x^2$$

How many barrels should be refined for maximum daily profit, and what is the maximum profit?

Question 7

[6 marks]

The rate of change of the temperature T (in $^{\circ}\text{C}$) from the center of a blast furnace to a distance s (in m) from the center is given by

$$\frac{dT}{ds} = -\frac{1500}{(s+1)^4}$$

Express T as a function of s if $T = 3000^{\circ}\text{C}$ for $s = 0$.

Question 8

[7 marks]

Determine the approximate value of the integral

$$\int_0^1 \sqrt{x^2 + 1} \, dx$$

By using the trapezoidal rule. Take $n = 5$ and write your answer to 3 decimal places.

Formula Sheet

1. Definition of Derivative:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

2. $\frac{d}{dx}(x^n) = nx^{n-1}$

3. Product rule:

$$\frac{d}{dx}(u v) = u \frac{dv}{dx} + v \frac{du}{dx}$$

4. Quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

5. Equation of Tangent line

$$y - y_1 = m(x - x_1)$$

6. Equation of Normal line

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

7. Trapezoidal rule

$$\int_a^b f(x) dx \approx \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$$

8. Simpson's rule

$$\int_a^b f(x) dx \approx \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots + 4y_{n-1} + y_n)$$

MLO & Bloom's Level of Complexity

Q #	MLO Addressed	Complexity Level	Mark	Remark
2, 6	2, 3, 5	Application	9	
4, 8	1,	Understanding/ Analysing	11	
3, 7	4	Evaluating	10	
1	2	Analysing	5	
5	1, 3	Remembering	5	
XX				
XX				
XX				
XX				