

## ACADEMIC YEAR 2023 - 2024

| Program            | Year               | Semester   | Paper             |
|--------------------|--------------------|------------|-------------------|
| <b>DO</b>          | <b>2</b>           | <b>2</b>   | <b>MAIN</b>       |
| MODULE NAME:       | <b>MATH 3</b>      |            |                   |
| MODULE CODE:       | <b>DMATH-III</b>   | EXAM DATE: | <b>21.05.2024</b> |
| INSTRUCTOR's NAME: | <b>Dr. Taofeek</b> | DURATION:  | <b>2 hrs.</b>     |

|   |   |   |
|---|---|---|
| <b>Questions to be answered on:</b><br><input checked="" type="checkbox"/> Space provided on the question paper | <b>Allowed tools:</b><br>Pen, Pencil & Calculator | <b>Number of pages</b><br>(Incl. cover page): <b>12</b> |
|---|---|---|

### Points of attention:

- For each question, the maximum earned points are mentioned between brackets at the end of each question.
- Write very clearly! Answers that are not readable are not marked and don't get points!
- Make sure your answers are written to the point.
- All answers should be written **in English**.
- Write all the answers in **blue or black pen only**.
- Use the **pencil** only for **diagrams & graphs**.
- Show all the calculation steps in the given space.
- When finished submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

| <b>FINAL MARKS</b> |  |           |
|--------------------|--|-----------|
| STUDENT NAME:      |  | <b>40</b> |
| STUDENT ID:        |  | <b>10</b> |

Number of answer scripts:.....

Invigilator:.....

Student's signature: .....

Time of receipt:.....

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**ANSWER ALL THE QUESTIONS**

**Question 1**

**[5 Marks]**

If the function  $f(x) = 3x + \frac{4}{x}$ , then

a) Find the derivative of  $f(x)$  using the definition.

(3 marks)

b) Find the slope of a line tangent to the curve  $f(x)$  at the point (2, 3). (2 marks)

**Question 2**

**[4 Marks]**

a. A bullet is fired vertically upward. Its distance  $s$  (in  $ft$ ) above the ground is given by (2 marks)

$$s = 2250t - 16t^2$$

where  $t$  is the time (in  $secs$ ). Determine the acceleration of the bullet.

b. A load (in  $N$ ) is distributed along a beam  $10 m$  long such that  $L = 5x - 0.5x^2$ , where  $x$  is the distance from one end of the beam. Find the expression for the instantaneous rate of change of  $L$  with respect to  $x$ ? (2 marks)

**Question 3**

**[4 marks]**

Evaluate the derivative of  $y = \frac{3x^2+x}{1-4x}$  at  $(2, -2)$

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**Question 4****[4 Marks]**

Evaluate the second derivative of the given function for the given value of  $x$ .

$$f(x) = 8(1 + 2x)^3, x = \frac{1}{4}$$

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**Question 5****[5 Marks]**

For the implicit function

$$3x^2 + xy + 2y^2 + 6 = 0$$

a) Find the slope of a line tangent to the curve of the function at the point (1, 2). (3 marks)

b) Find the equation of the normal line to the function at the point (1, 2). (2 marks)

**Question 6****[5 marks]**

A small oil refinery estimates that its daily profit  $P$  (in dollars) from refining  $x$  barrels of oil is

$$P = 8x - 0.02x^2$$

How many barrels should be refined for maximum daily profit, and what is the maximum profit?

**Question 7**

**[6 marks]**

The rate of change of the temperature  $T$  (in  $^{\circ}\text{C}$ ) from the center of a blast furnace to a distance  $s$  (in m) from the center is given by

$$\frac{dT}{ds} = -\frac{1500}{(s+1)^4}$$

Express  $T$  as a function of  $s$  if  $T = 3000^{\circ}\text{C}$  for  $s = 0$ .

**Question 8**

**[7 marks]**

Determine the approximate value of the integral

$$\int_0^1 \sqrt{x^2 + 1} \, dx$$

By using the trapezoidal rule. Take  $n = 5$  and write your answer to 3 decimal places.

### Formula Sheet

1. Definition of Derivative:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$2. \frac{d}{dx}(x^n) = nx^{n-1}$$

3. Product rule:

$$\frac{d}{dx}(u \ v) = u \frac{dv}{dx} + v \frac{du}{dx}$$

4. Quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

5. Equation of Tangent line

$$y - y_1 = m(x - x_1)$$

6. Equation of Normal line

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

7. Trapezoidal rule

$$\int_a^b f(x) dx \approx \frac{h}{2} (y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$$

8. Simpson's rule

$$\int_a^b f(x) dx \approx \frac{h}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots + 4y_{n-1} + y_n)$$

### MLO & Bloom's Level of Complexity

| Q #  | MLO Addressed | Complexity Level         | Mark | Remark |
|------|---------------|--------------------------|------|--------|
| 2, 6 | 2, 3, 5       | Application              | 9    |        |
| 4, 8 | 1,            | Understanding/ Analysing | 11   |        |
| 3, 7 | 4             | Evaluating               | 10   |        |
| 1    | 2             | Analysing                | 5    |        |
| 5    | 1, 3          | Remembering              | 5    |        |
| XX   |               |                          |      |        |