الجامعة الوطنية
National University
Science & Technology للعلوم والتكنولوجيا
كلية عمان البحرية الدولية
International Maritime College Oman

ACADEMIC YEAR 2023 - 2024

Program	Semester	Term	Paper
FOUNDATION	1	1	MAIN

MODULE NAME:	**PURE MATHEMATICS**		
MODULE CODE:	**FMTH005**	**EXAM DATE:**	**20/11/2023**
INSTRUCTOR's NAME:	**Muhammad Kazam**	**DURATION:**	**2 hrs.**

Questions to be answered on:	**Allowed tools:**	**Number of pages**
✓ Space provided on the question paper	Pen, Pencil & Calculator	(Incl. cover page): **11**

Points of attention:

- For each question, the maximum earned points are mentioned between brackets at the end of each question.
- Write very clearly! Answers that are not readable are not marked and don't get points!
- Make sure your answers are written to the point.
- All answers must be written **in English**.
- Write all the answers **in blue or black pen only**.
- When finished, submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Cheating / copying is not allowed and will result in failing the exam.

STUDENT NAME:	
STUDENT ID:	
CLASS:	

FINAL MARKS

	10
	40

Number of answer scripts:……..…….. **Invigilator:**……..……..…..

Student's signature: …… **Time of receipt:**……..…..

Question 1 [4 Marks]

Samples of a cast aluminum part are classified on the basis of surface finish in microinches and length measurements. The results are summarized as follows:

		Length	
		Excellent	Good
Surface Finish	Excellent	80	2
	Good	10	8

Let A denote the event that a sample has excellent surface finish, and let B denote the event that a sample has excellent length. Determine the probabilities for the events given below.

a. $P(A)$ **(1 mark)**

b. $P(B')$ **(1 mark)**

c. $P(A \cup B)$ **(2 marks)**

Question 2 [5 Marks]

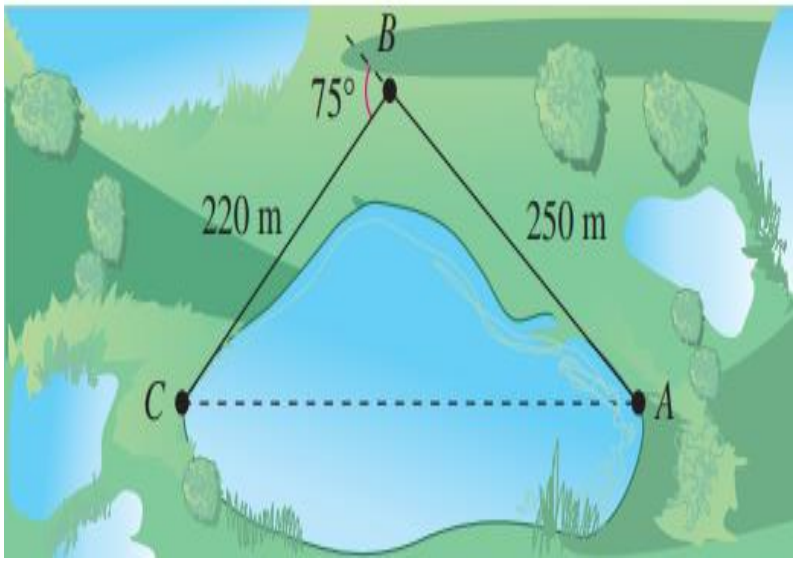
The following data give the worldwide number of fatal airline accidents of commercially scheduled air transports in the years from 1985 to 1993.

22 22 26 28 27 25 30 29 24

Calculate the mean and variance for the above data.

Question 3 [3 Marks]

To approximate the length of a land, a surveyor walks 250 meters from point A to point B, then turns and walks 220 meters to point C (see figure). Approximate the length of the land.



Question 4 [5 Marks]

Solve the following equation.

$\mathbf{1 - \log(x - 2) = \log(3x + 1)}$.

Question 5 [5 Marks]

Prove that $\cos x - \frac{\cos x}{1 - \tan x} = \frac{\cos x \sin x}{\sin x - \cos x}$.

Question 6 [5 Marks]

Bacteria population that experiences exponential growth increases according to the model $\boldsymbol{n(t) = n_0 e^{rt}}$, where $n(t)$ is the population at time t, n_0 is the initial population and r is the relative growth rate.

The initial bacteria count in a culture is 500 and the relative growth rate is 40% per hour.

a. Find a function that models the number of bacteria after t hours. **(1 mark)**

b. Calculate the bacteria count after 10 hours. **(1 mark)**

c. After how many hours will the bacteria count reach 80000? **(3 marks)**

Question 7 [5 Marks]

Calculate the inverse of the function $\boldsymbol{f(x) = \frac{2x+5}{3x-4}}$.

Question 8 [8 Marks]

The speed at which a car is driven can have a big effect on fuel mileage. Based on the information, the function $\boldsymbol{m(x) = -0.025x^2 + 2.45x - 30}$, where $30 \leq x \leq 65$, models the average miles per gallon for cars in terms of the speed driven x (in miles per hour).

a. Calculate the miles a car should drive to maximize mileage per gallon. **(3 marks)**

b. Draw the graph for the function $\boldsymbol{m(x)}$. **(5 marks)**

Formula Sheet

Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Law of Sines	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Law of Cosines	$a^2 = b^2 + c^2 - 2bc\ cosA$ $b^2 = a^2 + c^2 - 2ac\ cosB$ $c^2 = a^2 + b^2 - 2ab\ cosC$
Varinace	$s^2 = \frac{1}{n-1}\left[\sum X^2 - \frac{(\sum X)^2}{n}\right]$

References:

Stewart, J., Redlin, L. and Watson, S. (2017) *Precalculus Mathematics for Calculus*. 7th edn. Cengage.