

## FINAL EXAM

### MMATH-II: MATHEMATICS-2

### Spring – 2025

#### Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit and sign the question paper to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration: 2 hours  
Instructor's Name: Muhammad Kazam  
Exam Date: 18/06/2025  
Program: ME

	<b>40</b>
	<b>10</b>

#### Student Information

Name:  ID:   
Signature:

#### Invigilator

Initials:  ☐ Student ID checked  
Time received:

**Question 1****[4 Marks]**

Three vectors  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are given below.

$$\mathbf{u} = \mathbf{i} - \mathbf{j} + \mathbf{k}, \quad \mathbf{v} = -\mathbf{j} + \mathbf{k}, \quad \mathbf{w} = \mathbf{i} + \mathbf{j} + \mathbf{k}$$

a. Determine the scalar triple product  $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ .

**(3 marks)**

b. State whether the vectors are coplanar or not.

**(0.5 mark)**

c. If not, find the volume of the parallelepiped that they determine.

**(0.5 mark)**

**Question 2****[3 Marks]**

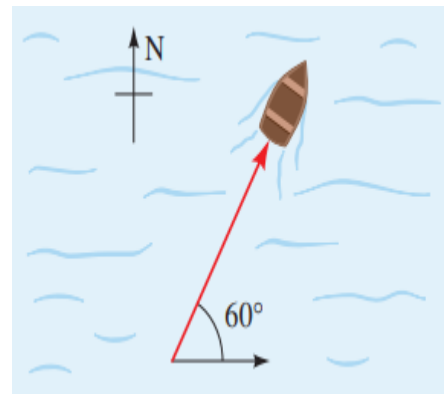
Find the area of  $\Delta PQR$  whose vertices are given below.

$P(3, -2, 6), Q(-1, -4, -6), R(3, 4, 6)$

**Question 3****[8 Marks]**

A straight river flows east at a speed of  $10 \text{ km/h}$ . A boater starts at the south shore of the river and head in a direction  $60^\circ$  from the shore as shown in the figure. The motorboat has a speed of  $20 \text{ km/h}$  relative to the water.

a. Express the velocity of the river as a vector in component form.

**(2 marks)**

(Stewart, Redlin and Watson, 2017)

**b.** Express the velocity of the motorboat relative to the water as a vector in component form.

**(2 marks)**

**c.** Determine the true velocity of the motorboat.

**(1 mark)**

**d.** Determine the true speed and direction of the motorboat.

**(3 marks)**

**Question 4****[3 Marks]**

For the function given below, determine each limit, if it exists. If it does not exist, explain why.

$$f(x) = \begin{cases} 2 & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x \leq 2 \\ x + 2 & \text{if } x > 2 \end{cases}$$

a.  $\lim_{x \rightarrow -1^1} f(x)$

b.  $\lim_{x \rightarrow -1^+} f(x)$

c.  $\lim_{x \rightarrow -1} f(x)$

d.  $\lim_{x \rightarrow 2} f(x)$

e.  $\lim_{x \rightarrow 0} f(x)$

f.  $\lim_{x \rightarrow 3} [f(x)]^2$

**Question 5****[10 Marks]**

Determine the limit, if it exists.

a.  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x}$

**(3 marks)**

b.  $\lim_{x \rightarrow \infty} \left( \frac{1}{x} - \frac{2x}{x-1} \right)$

**(3 marks)**

c.  $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x^2 + x - 2}$

**(4 marks)**

**Question 6****[5 Marks]**

Calculate the inverse of the matrix given below.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 2 & 5 & 6 \end{bmatrix}$$





**Question 7****[7 Marks]**

Solve the following system of equations by Gauss method or Cramer's rule.

$$\begin{cases} 2x + 3z = 5 \\ x + y + 6z = 0 \\ 3x - y + z = 5 \end{cases}$$



**MLO & Bloom's Level of Complexity**

Q #	MLO Addressed	Complexity Level	Mark	Remark
1	1 & 2	Understanding	4	Expect 100% to solve
2	3	Understanding	3	Expect 90% to solve
3	2 & 4	Application	8	Expect 100% to solve
4	5	Evaluating /Application	3	Expect 80% to solve
5	1, 2 & 5	Understanding / Application	10	Expect 90% to solve
6	4	Evaluating /Application	5	Expect 80% to solve
7	4 & 5	Application	7	Expect 70% to solve

**Reference:**

Stewart, J., Redlin, L. and Watson, S. (2017) *Precalculus Mathematics for Calculus*. 7th edn. Cengage.