

Final Exam
MMATH-III: MATH 3
Fall 2025

Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration: 2 hours
Instructor's Name: Dr. Rokhsaneh Yousef Zehi
Exam Date: 29/12/2025
Program: ME

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	10

Student Information	
Name:	<input type="text"/>
Signature:	<input type="text"/>
ID:	<input type="text"/>

Invigilator	
Initials:	<input type="text"/>
Time received:	<input type="text"/>
<input type="checkbox"/> Student ID checked	

Question 1**[5 marks]**

Given the following implicit function

$$y^3 + 3xy^2 = x^2 + 1$$

- (a) Determine the slope of the tangent line at the point $(1, -1)$. (3 marks)
- (b) Obtain the equation of the normal line to the curve at the point $(1, -1)$. (2 marks)

Question 2**[3 marks]**

Water is being drained from a spherical tank at a rate of $0.6 \text{ m}^3/\text{min}$.

(a) How fast is the radius of the water decreasing when the radius is 5 m . (2 marks)

(b) Determine the rate of change of surface area of the water when the radius is 5 m .

(1 marks)

Question 3**[7 marks]**

A rectangular container is required to have a volume of 200 cm^3 . The container has a length of $x \text{ cm}$, a width of $2x \text{ cm}$, and a height of $y \text{ cm}$. To reduce material use, the aim is to construct the container with the smallest possible surface area.

Determine the dimensions (length, width, and height) that will minimize the surface area.

Question 4**[5 marks]**

An engineering team is measuring how the stress S (in Mpa) on a beam change as an applied load of x kilonewtons is added. They determine that

$$\frac{dS}{dx} = \frac{20x}{\sqrt{100 + x^2}}$$

Determine the total change in stress S as the load increases from $x = 0$ KN to $x = 40$ KN .

Question 5**[5 marks]**

Approximate the value of

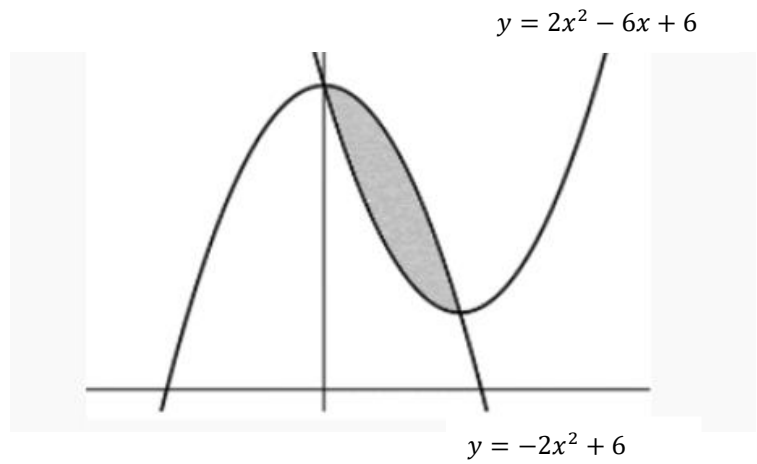
$$\int_1^4 \frac{12}{x^2 + 9} dx$$

using **Simpson's Rule** with $n = 6$. Give your answer correct to 3 decimal places.

Question 6**[5 marks]**

The diagram below shows the graphs of $y = -2x^2 + 6$ and $y = 2x^2 - 6x + 6$. A region is shaded between the two curves, beginning and ending at their points of intersection.

Determine the area of the shaded region.



Question 7**[4 marks]**

A fuel tank on a small marine engine boat is formed by revolving the curve

$$y = \frac{1}{3}x\sqrt{5 - x^2}$$

around the x -axis, where y is the height (in m) and x is the horizontal distance (in m).

Determine the volume of the tank formed by revolving this curve from $x = 0$ to $x = \sqrt{5}$.

Question 8**[6 marks]**

A cylindrical tank with a hole at the bottom is filled with water to a height h_0 . The water flows out due to gravity. According to Torricelli's Law, the rate at which the water height decreases is proportional to the square root of the height of the water column.

$$\frac{dh}{dt} = -k\sqrt{h}$$

Where h is the height of water in the tank (in m) at time t (in min), and k is a constant related to the size of the hole and gravitational acceleration.

- (a) Determine the general solution for h . (4 marks)
- (b) Apply the conditions $h(0) = 4 m$ and $k = 0.1$ and determine the height of the water in the tank at $t = 10$ minutes. (2 marks)

This page is for rough work.

Formula sheet:

Product Rule	$(uv)' = u'v + v'u$
Quotient Rule	$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$
Volume of a Sphere	$V = \frac{4}{3}\pi r^3$
Surface area of a Sphere	$A = 4\pi r^2$
Volume of a Rectangular box	$V = l \times w \times h$
Surface area of a Rectangular box	$A = 2(lw + lh + wh)$
Simpson's rule	$\int_a^b f(x)dx \approx \frac{h}{3}[y_0 + 4y_1 + 2y_2 + \dots + 4y_{n-1} + y_n]$

MLO and Bloom's Level of Complexity

Q #	MLO Addressed	Complexity Level	Mark	Remark
1	1,2	Application	5	
2	2,3	Application, Analysis	3	
3	3,4	Analysis, Evaluate	7	
4	1,2	Application	5	
5	1	Application	5	
6	1	Application	5	
7	2,3	Application, Analysis	4	
8	2,3	Application, Analysis	6	

References:

1. J. Washington, A., 2014. Basic Technical Mathematics with Calculus. 10 ed. Harlow: Pearson Education Limited.
2. Stewart, J., 2008. *Calculus: Early Transcendentals*. 6th ed. Boston: Brooks/Cole.