

Final Exam
MMATH-III: MATH 3
Spring 2025

Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration:

2.5 hrs.

Instructor's Name:

Dr. Taofeek Olanrewaju Alade

Exam Date:

23/06/2025

Program:

ME

	40

Student Information

Name:

ID:

Signature:

Invigilator

Initials:

Student ID checked

Time received:

ANSWER ALL THE QUESTIONS**Question 1****[4 marks]**

The electric power E produced by a certain source is given by

$$E = \frac{V^2 r}{R^2 + 2Rr + r^2}$$

Where V is the voltage of the source, R is the resistance of the source, and r is the resistance in the circuit. Find the derivative of E with respect to r , assuming that the other quantities remain constant.

Question 2**[4 marks]**

For the implicit function

$$3xy^2 + x^2 + y^2 = 8$$

a) Determine the slope of a line tangent to the curve of the function at the point (1, 2).
(2 marks)

b) Determine the equation of the normal line to the function at the point (1, 2).
(2 marks)

Question 3**[4 marks]**

The function $f(x)$ is given by

$$f(x) = 8x^3 - 12x^2 - 6$$

(a) Determine the values of x for which $\frac{df}{dx} = 0$. (2 marks)

(b) Determine the values of x for which $\frac{d^2f}{dx^2} = 0$. (2 marks)

Question 4**[5 marks]**

Evaluate the definite integral

$$\int_2^4 x \left(\sqrt{(x^2 + 1)^3} \right) dx$$

Question 5**[4 marks]**

A small oil refinery estimates that its daily profit P (in dollars) from refining x barrels of oil is

$$P = 18x - 0.08x^2$$

How many barrels should be refined for maximum daily profit, and what is the maximum profit?

Question 6**[7 marks]**

Determine the approximate value of the integral

$$\int_{1}^{4} (2x + \sqrt{x}) \, dx$$

By using the Simpson's rule. Take $n = 6$ and write your answer to 3 decimal places.

Question 7.**[5 marks]**

A curved section of a ship's hull follows the profile given by the equation

$$y = x^3 - 5x^2 + 4x$$

where x and y are measured in meters. Calculate the total area enclosed between this curve and the x axis from $x = 0$ to $x = 3$. This area represents the submerged cross-sectional area of the hull.

Question 8.**[7 marks]**

a. Verify that $y = A \cos x + B \sin x$ satisfies the differential equation

$$\frac{d^2y}{dx^2} + y = 0 \quad (3 \text{ marks})$$

b. The general solution of $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 0$ is $y = A e^t + B e^{2t}$. Find the particular solution that satisfies $y = 3$ and $\frac{dy}{dt} = 5$, when $t = 0$. (4 marks)

Formula Sheet

1. Definition of Derivative:

$$f^1(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$2. \frac{d}{dx}(x^n) = nx^{n-1}$$

3. Product rule:

$$\frac{d}{dx}(u v) = u \frac{dv}{dx} + v \frac{du}{dx}$$

4. Quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

5. Equation of Tangent line

$$y - y_1 = m(x - x_1)$$

6. Equation of Normal line

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

7. Trapezoidal rule

$$\int_a^b f(x) dx \approx \frac{h}{2} (y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$$

8. Simpson's rule

$$\int_a^b f(x) dx \approx \frac{h}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots + 4y_{n-1} + y_n)$$

MLO & Bloom's Level of Complexity

Q #	MLO Addressed	Complexity Level	Mark	Remark
1, 3, 5	2, 3, 5	Application	12	
6	1,	Understanding/ Analysing	7	
4	4	Evaluating	5	
2, 7	2	Analysing	9	
8	1, 3	Remembering	7	