

Final Exam
MMATH-III: MATH 3
Spring 2025

Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration: 2.5 hrs.
Instructor's Name: Dr. Taofeek Olanrewaju Alade
Exam Date: 23/06/2025
Program: ME

	40

Student Information

Name: ID:
Signature:

Invigilator

Initials: ☐ Student ID checked
Time received:

ANSWER ALL THE QUESTIONS**Question 1****[4 marks]**

The electric power E produced by a certain source is given by

$$E = \frac{V^2 r}{R^2 + 2Rr + r^2}$$

Where V is the voltage of the source, R is the resistance of the source, and r is the resistance in the circuit. Find the derivative of E with respect to r , assuming that the other quantities remain constant.

Question 2**[4 marks]**

For the implicit function

$$3xy^2 + x^2 + y^2 = 8$$

- a) Determine the slope of a line tangent to the curve of the function at the point (1, 2).
(2 marks)
- b) Determine the equation of the normal line to the function at the point (1, 2).
(2 marks)

Question 3**[4 marks]**

The function $f(x)$ is given by

$$f(x) = 8x^3 - 12x^2 - 6$$

- (a) Determine the values of x for which $\frac{df}{dx} = 0$. (2 marks)

- (b) Determine the values of x for which $\frac{d^2f}{dx^2} = 0$. (2 marks)

Question 4**[5 marks]**

Evaluate the definite integral

$$\int_2^4 x \left(\sqrt{(x^2 + 1)^3} \right) dx$$

Question 5**[4 marks]**

A small oil refinery estimates that its daily profit P (in dollars) from refining x barrels of oil is

$$P = 18x - 0.08x^2$$

How many barrels should be refined for maximum daily profit, and what is the maximum profit?

Question 6**[7 marks]**

Determine the approximate value of the integral

$$\int_1^4 (2x + \sqrt{x}) \, dx$$

By using the Simpson's rule. Take $n = 6$ and write your answer to 3 decimal places.

Question 7.**[5 marks]**

A curved section of a ship's hull follows the profile given by the equation

$$y = x^3 - 5x^2 + 4x$$

where x and y are measured in meters. Calculate the total area enclosed between this curve and the x axis from $x = 0$ to $x = 3$. This area represents the submerged cross-sectional area of the hull.

Question 8.**[7 marks]**

- a. Verify that $y = A \cos x + B \sin x$ satisfies the differential equation

$$\frac{d^2 y}{dx^2} + y = 0$$

(3 marks)

- b.** The general solution of $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 0$ is $y = A e^t + B e^{2t}$. Find the particular solution that satisfies $y = 3$ and $\frac{dy}{dt} = 5$, when $t = 0$. (4 marks)

Formula Sheet

1. Definition of Derivative:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

2. $\frac{d}{dx}(x^n) = nx^{n-1}$

3. Product rule:

$$\frac{d}{dx}(u v) = u \frac{dv}{dx} + v \frac{du}{dx}$$

4. Quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

5. Equation of Tangent line

$$y - y_1 = m(x - x_1)$$

6. Equation of Normal line

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

7. Trapezoidal rule

$$\int_a^b f(x) dx \approx \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$$

8. Simpson's rule

$$\int_a^b f(x) dx \approx \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots + 4y_{n-1} + y_n)$$

MLO & Bloom's Level of Complexity

Q #	MLO Addressed	Complexity Level	Mark	Remark
1, 3, 5	2, 3, 5	Application	12	
6	1,	Understanding/ Analysing	7	
4	4	Evaluating	5	
2, 7	2	Analysing	9	
8	1, 3	Remembering	7	