

Final Exam
MMATH-III: MATH 3
Spring 2025

Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration:

2.5 hours

Instructor's Name:

Dr. Taofeek Olanrewaju Alade

Exam Date:

23/06/2025

Program:

ME

	40

Student Information

Name:

ID:

Signature:

Invigilator

Initials:

Student ID checked

Time received:

ANSWER ALL THE QUESTIONS**Question 1****[4 marks]**

For the implicit function

$$4xy^2 + x^2y + y^2 - 4 = 0$$

- a) Determine the slope of a line tangent to the curve of the function at the point $(-1, 2)$.
(2 marks)
- b) Determine the equation of the normal line to the function at the point $(-1, 2)$.
(2 marks)

Question 2**[5 marks]**

If the function $f(x) = 3x^3 + \frac{4}{x}$, then

a) Find the derivative of $f(x)$ using the definition. (3 marks)

b) Find the slope of a line tangent to the curve $f(x)$ at the point $(2, 3)$. (2 marks)

Question 3**[4 marks]**

The rate of change of the temperature T (in $^{\circ}\text{C}$) from the center of a blast furnace to a distance s (in m) from the center is given by

$$\frac{dT}{ds} = -\frac{1500}{(s+1)^4}$$

Express T as a function of s if $T = 3000^{\circ}\text{C}$ for $s = 0$.

Question 4**[5 marks]**

Evaluate the definite integral

$$\int_1^4 y \left(\sqrt{(2 + y^2)^3} \right) dy$$

Question 5**[4 marks]**

A coastal marine fuel processing station estimates that its daily profit P (in dollars) from refining x barrels of marine diesel fuel is modeled by the equation:

$$P = 15x - 0.5x^2$$

As a marine engineer overseeing operational efficiency, determine:

- a. The optimal number of barrels that should be refined daily to achieve maximum profit. (2 marks)
- b. The corresponding maximum daily profit. (2 marks)

Question 6**[6 marks]**

Determine the approximate value of the integral

$$\int_1^4 (\sqrt{x} + 2x^2) dx$$

By using the trapezoidal rule. Take $n = 6$ and write your answer to 3 decimal places.

Question 7.**[5 marks]**

A section of a pipe used in a marine fuel system is shaped by rotating the area between the line $y = 2 - 2x$ and the y-axis, from $y = -2$ to $y = 2$, around the y-axis. Determine the volume of the pipe section formed by this rotation.

Question 8.**[7 marks]**

a. Verify that $x = 3 \sin 2t$ is a solution of

$$\frac{d^2x}{dt^2} + 4x = 0 \quad (3 \text{ marks})$$

b. The general solution of $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = 0$ is $x = Ate^t + Be^t$. Determine the particular solution satisfying $x(0) = 0$ and $\frac{dx}{dt}(0) = 1$. (4 marks)

Formula Sheet

1. Product rule:

$$\frac{d}{dx}(u v) = u \frac{dv}{dx} + v \frac{du}{dx}$$

2. Quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

3. Equation of Tangent line

$$y - y_1 = m(x - x_1)$$

4. Equation of Normal line

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

5. Trapezoidal rule

$$\int_a^b f(x) dx \approx \frac{h}{2} (y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$$

6. Simpson's rule

$$\int_a^b f(x) dx \approx \frac{h}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots + 4y_{n-1} + y_n)$$

MLO & Bloom's Level of Complexity

Q #	MLO Addressed	Complexity Level	Mark	Remark
1, 3, 5	2, 3, 5	Application	12	
6	1,	Understanding/ Analysing	6	
4	4	Evaluating	5	
2, 7	2	Analysing	10	
8	1, 3	Remembering	7	

