الجامعة الوطنية
National University
Science & Technology للعلوم والتكنولوجيا
كلية عمان البحرية الدولية
International Maritime College Oman

ACADEMIC YEAR 2023 - 2024

Program	Year	Semester	Paper
PE-PTB	**2**	**1**	**MAIN**

MODULE NAME:	**APPLIED MATHS FOR PROCESS ENGINEERING**		
MODULE CODE:	**PT-TMATH-III**	**EXAM DATE:**	**31-12-2023**
INSTRUCTOR's NAME:	**Ranjit V**	**DURATION:**	**2 hrs 30 mins**

Questions to be answered on:	**Allowed tools:**	**Number of pages**
✓ Space provided on the question paper	Pen, Pencil & Calculator	(Incl. cover page): **14**

Points of attention:

- For each question, the maximum earned points are mentioned between brackets at the end of each question.
- Write very clearly! Answers that are not readable are not marked and don't get points!
- Make sure your answers are written to the point.
- All answers should be written **in English.**
- Write all the answers in **blue or black pen only.**
- Use the **pencil** only for **diagrams & graphs.**
- Show all the calculation steps in the given space.
- When finished submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

FINAL MARKS

	40
	10

STUDENT NAME:

STUDENT ID:

Number of answer scripts:………………………. **Invigilator:**……………………………………

Student's signature: ……………………………. **Time of receipt:**…………………………......

INSTRUCTIONS

Answer ***any four questions*** *only from the five questions provided. If you do all the questions, the first four will be only marked. Each question carries 10 marks.*

ANSWER THE QUESTIONS IN THE SPACE PROVIDED

1. a) A digital encoding system transforms a message in terms of a number system using the following equation matrix **(4 marks)**

$$\begin{pmatrix} 3 & -1 \\ 1 & a \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} b & d \\ 12 & c \end{pmatrix}$$

Determine the value of a, b, c and d.

b) Determine the eigenvalues and associated eigenvectors of the matrix given below: **(6 marks)**

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$$

2. Evaluate the following expressions given below using the table of Laplace transform in the formulae sheet

a) Laplace transform of **(4 marks)**

$$2t\sin 3\pi t + e^{-2t}\cos 4t + 6t^3 e^{5t} + t^3$$

b) Laplace inverse transform of **(6 marks)**

$$\frac{3 - 2s}{s^2 + 4s + 8}$$

3. The pressure vs temperature data measured using a reator experiment si given below:

Temperature T(K)	87.5	84	77.8	63.7	46.7
Pressure P (kPa)	292	283	270	235	197

Determine a second degree parabolic curve of the form $P = a\,T^2 + b\,T + c$ to fit the given data.

(10 marks)

4. a) The production of refined oils (in barrels) from crude oil in Sohar refinery for the last 20 years is given below:

Year (x)	2007	2011	2015	2019	2023
Production in thousand of barrel per year(y)	250	284	323	306	223

Estimate the production of oil in the year of 2022. **(5 marks)**

Marks	30-40	40-50	50-60	60-70	70-80

b) In an examination the number of candidates who obtained marks between certain limits were as follows

No. of Students	31	42	51	35	31

Calculate the number of candidates whose scores lie between 45 and 50. **(5 marks)**

5. The pressure P, of a gas, varies with temperature, t according to the following 2nd-order differential equation

$$P'' - P' + P = t \qquad where\ P'' - \frac{d^2P}{dt^2}\ \&\ P' - \frac{dP}{dx}$$

The initial boundary condition are given as P (0) = 1 and P'(0) = 1.

Solve the equation to determine the pressure in terms of t using Laplace and inverse Laplace transform.

(10 marks)

Formulae Sheet

1.	the normal equations fitting to a straight line $$a\sum x^2 + b\sum x = \sum xy$$ and $$a\sum x + bn = \sum y$$
2.	the normal equations fitting to a linear second-order degree curve $$a\sum x_i^4 + b\sum x_i^3 + c\sum x_i^2 = \sum x_i^2 y_i$$ $$a\sum x_i^3 + b\sum x_i^2 + c\sum x_i = \sum x_i y_i$$ $$a\sum x_i^2 + b\sum x_i + cn = \sum y_i$$
3.	Newton's forward interpolation formulae $$y_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 \cdots\cdots + \frac{p(p-1)\cdots\cdots(p-n+1)}{n!}\Delta^n y_0$$
4.	Newton’s backward interpolation formuala $$y_n(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n \cdots\cdots + \frac{p(p+1)\cdots\cdots(p+n-1)}{n!}\nabla^n y_n$$
5.	Newton’s divided difference interpolation formula $$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0)$$ $$\cdots\cdots + (x-x_0)(x-x_1)(x-x_2)\cdots\cdots(x-x_{n-1})\Delta^n f(x_0)$$
6.	Lagrange’s interpolation formula $$f(x) = \frac{(x-x_1)(x-x_2)\cdots\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots\cdots(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)\cdots\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots\cdots(x_1-x_n)} f(x_1) + \frac{(x-x_0)(x-x_1)\cdots\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)\cdots\cdots(x_2-x_n)} f(x_2)$$ $$\cdots + \frac{(x-x_0)(x-x_1)\cdots\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots\cdots(x_n-x_{n-1})} f(x_n)$$

Table of Laplace Transforms

	$f(t)=\mathcal{L}^{-1}(F)$	$\mathcal{L}(f)=F(s)$		$f(t)=\mathcal{L}^{-1}(F)$	$\mathcal{L}(f)=F(s)$
1.	1	$\frac{1}{s}$	11.	te^{-at}	$\frac{1}{(s+a)^2}$
2.	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}(n=1,2,3,\dots)$	12.	$t^{n-1}e^{-at}$	$\frac{(n-1)!}{(s+a)^n}$
3.	e^{-at}	$\frac{1}{s+a}$	13.	$e^{-at}(1-at)$	$\frac{s}{(s+a)^2}$
4.	$1-e^{-at}$	$\frac{a}{s(s+a)}$	14.	$[(b-a)t+1]e^{-at}$	$\frac{s+b}{(s+a)^2}$
5.	$\cos at$	$\frac{s}{s^2+a^2}$	15.	$\sin at - at\cos at$	$\frac{2a^3}{(s^2+a^2)^2}$
6.	$\sin at$	$\frac{a}{s^2+a^2}$	16.	$t\sin at$	$\frac{2as}{(s^2+a^2)^2}$
7.	$1-\cos at$	$\frac{a^2}{s(s^2+a^2)}$	17.	$\sin at + at\cos at$	$\frac{2as^2}{(s^2+a^2)^2}$
8.	$at-\sin at$	$\frac{a^3}{s^2(s^2+a^2)}$	18.	$t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
9.	$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$	19.	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
10.	$ae^{-at}-be^{-bt}$	$\frac{s(a-b)}{(s+a)(s+b)}$	20.	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$

The differentiation property of the Laplace Transform

$$\frac{df(t)}{dt} \xleftrightarrow{\mathcal{L}} s\,F(s)-f(0)$$

$$\frac{d^2f(t)}{dt^2} \xleftrightarrow{\mathcal{L}} s^2\,F(s)-s\,f(0)\;-f'(0)$$