National University Science & Technology
International Maritime College Oman

Final Exam

PT-TMATH-III-I: APPLIED MATHS FOR PROCESS ENGINEERING

Fall 2025

Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration: 3 hours
Instructor's Name: Dr. Rokhsaneh Yousef Zehi
Exam Date: 04/01/2026
Program: PE

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Student Information

Name: ____________ ID: ____________

Signature: ____________

Invigilator

Initials: ____________ ☐ Student ID checked

Time received: ____________

Question 1 **[8 marks]**

A thermal system consists of three interconnected metal plates whose temperature deviations from ambient (in °C) are denoted by $T_1(t)$, $T_2(t)$, $T_3(t)$. The heat exchange between the plates is described by the following system of differential equations:

$$\boldsymbol{\frac{dT_1}{dt} = 5T_1 - T_2 + T_3}$$

$$\boldsymbol{\frac{dT_2}{dt} = 4T_2 - 2T_3}$$

$$\boldsymbol{\frac{dT_3}{dt} = T_2 + T_3}$$

where "t" is time in minutes.

Using the **eigenvalue and eigenvector method**, solve the system of differential equations to obtain expressions for $T_1(t)$, $T_2(t)$, $T_3(t)$.

Question 2 **[6 marks]**

(a) Determine the Laplace transform of the following function: (2 marks)

$$e^{4t}\sin 3t\,(2e^{-t}+3e^{-2t})$$

(b) Determine the inverse Laplace transform of the following function: (4 marks)

$$\frac{2s+3}{2s^2+8s+16}$$

Question 3 **[8 marks]**

An RLC series circuit has resistance R, inductance L, and capacitance C. The current $i(t)$ in the circuit satisfies the following differential equation:

$$\frac{d^2 i}{dt^2} + 7\frac{di}{dt} + 12i = 1$$

At time $t = 0$, the current and its rate of change are:

$$i(0) = -2, \qquad i'(0) = 5$$

Using the **Laplace transform method**, determine $i(t)$, the current in the circuit as a function of time.

Question 4 **[5 marks]**

A chemical reaction takes place in a batch reactor, and the temperature inside the reactor increases over time. The reactor has a cooling jacket to help keep the temperature under control. During the process, the reactor temperature $\boldsymbol{T}$ (℃) is measured at different times $\boldsymbol{t}$ $(\boldsymbol{min})$. The recorded data are:

Time $\boldsymbol{t}$ $(\boldsymbol{min})$	Temperature $\boldsymbol{T}$ (℃)
2	50
4	50.2
6	53.2
8	66.2
10	101.2
12	175
14	309.2

Using Newton's interpolation method, estimate the temperature at $t = 11\ min$.

Question 5 **[6 marks]**

Determine the first 4 non-zero terms of Maclaurin series expansion for $\boldsymbol{f(x) = x^2 \sin 2x}$.

Question 6 **[7 marks]**

Let

$$\boldsymbol{f(t)} = \begin{cases} \mathbf{0} & \mathbf{-3 < t < 0} \\ \mathbf{4} & \mathbf{0 < t < 3} \end{cases}$$

and suppose that $f(t)$ is periodic with period $\boldsymbol{T = 6}$, so that $f(t+6) = f(t)$.

Express the function $f(t)$ as a Fourier series and write down the first four non-zero terms of the series.

This page is for rough work.

Formula sheet

Table1. Common functions and their Laplace transform

Function	Laplace transform	Function	Laplace transform
$\mathbf{1}$	$\frac{1}{s}$	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
t^n	$\frac{n!}{s^{n+1}}$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
e^{at}	$\frac{1}{s-a}$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
e^{-at}	$\frac{1}{s+a}$	$t\,sos\,bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$\sinh t$	$\frac{b}{s^2-b^2}$
$\sin bt$	$\frac{b}{s^2+b^2}$	$\cosh t$	$\frac{s}{s^2-b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$		

Table 2. The Laplace transform of derivatives

$\mathcal{L}\{f'(t)\}$	$sF(s)-f(0)$
$\mathcal{L}\{f''(t)\}$	$s^2F(s)-sf(0)-f'(0)$
$\mathcal{L}\{f'''(t)\}$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

Newton's Interpolation	$y_n(x)=y_o+p\Delta y_0+\frac{p(p-1)}{2!}\Delta^2 y_0+\cdots+\frac{p(p-1)\ldots(p-n+1)}{n!}\Delta^n y_0$ $y_n(x)=y_n+p\nabla y_n+\frac{p(p+1)}{2!}\nabla^2 y_n+\cdots+\frac{p(p+1)\ldots(p+n-1)}{n!}\nabla^n y_n$

Fourier series	$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2n\pi t}{T} + b_n \sin\frac{2n\pi t}{T}\right)$$

MLO and Bloom's Level of Complexity

Q #	MLO Addressed	Complexity Level	Mark	Remark
1	**2, 3**	Application, Analysis	**8**	
2	**1**	Application	**6**	
3	**3, 4**	Application, Analysis	**8**	
4	**2,3**	Application, Analysis	**5**	
5	**1**	Application	**6**	
6	**1**	Application	**7**	

References:

1. Croft, T. and Davison, R. (2020). Mathematics for Engineers (Fifth Edition). UK: Pearson.
2. Loughborough University Mathematics Learning Support Centre. (n.d.). *HELM Workbooks*. Retrieved October 12, 2025, https://www.lboro.ac.uk/departments/mlsc/student-resources/helm-workbooks/