

Final Exam
**PT-TMATH-III-I: APPLIED MATHS FOR PROCESS
ENGINEERING**
Fall 2025

Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration: 3 hours
Instructor's Name: Dr. Rokhsaneh Yousef Zehi
Exam Date: 04/01/2026
Program: PE

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Student Information	
Name:	<input type="text"/>
ID:	<input type="text"/>
Signature:	<input type="text"/>

Invigilator	
Initials:	<input type="text"/>
Time received:	<input type="text"/>
<input type="checkbox"/> Student ID checked	

Question 1**[8 marks]**

Three continuously stirred reactors, R_1 , R_2 and R_3 are connected so that reactant flows between them. Let $r_1(t)$, $r_2(t)$ and $r_3(t)$ be the amounts of reactant (in mol/l) in reactors R_1 , R_2 and R_3 at time t (in minutes) respectively. After setting up the reaction process, the system of differential equations that describes the reactant amounts is:

$$\frac{dr_1}{dt} = r_1 + r_2 + r_3$$

$$\frac{dr_2}{dt} = -4r_2 + 2r_3$$

$$\frac{dr_3}{dt} = -3r_2 + r_3$$

Using the **eigenvalue and eigenvector method**, solve the system of differential equations to obtain expressions for $r_1(t)$, $r_2(t)$, $r_3(t)$.

Question 2**[6 marks]**

(a) Determine the Laplace transform of the following function:

(2 marks)

$$t(3t^2 e^{-t} + \cos 3t)$$

(b) Determine the inverse Laplace transform of the following function:

(4 marks)

$$\frac{4s + 2}{2s^2 + 4s + 10}$$

Question 3**[8 marks]**

In a long fluid pipeline, rapid changes in pump speed can create small pressure oscillations. The deviation in pressure from steady state is denoted by $p(t)$. For small disturbances, the pressure dynamics can be approximated by the following differential equation:

$$\frac{d^2p}{dt^2} + 5 \frac{dp}{dt} + 6p = 1$$

At $t = 0$, the pressure deviation and its rate of change are:

$$p(0) = 1, \quad p'(0) = 2$$

Using the **Laplace transform method**, determine $p(t)$, the pressure deviation as a function of time.

Question 4**[5 marks]**

In a stirred tank mixing process, the concentration of a dissolved chemical is measured at regular time intervals. The measured concentrations are shown in the table below:

Time <i>t (min)</i>	concentration <i>C (mol/l)</i>
0	7.0
2	7.4
4	9.0
6	13.0
8	20.6
10	33.0

Using Newton's interpolation method, estimate the concentration at $t = 3$ minutes.

Question 5**[6 marks]**

Determine the first 4 non-zero terms of Taylor series expansion for $f(x) = (x - 1) \ln x$ about the point $x = 1$.

Question 6**[7 marks]**

Let

$$f(t) = \begin{cases} -2 & -2 < t < 0 \\ 0 & 0 < t < 2 \end{cases}$$

and suppose that $f(t)$ is periodic with period $T = 4$, so that $f(t + 4) = f(t)$.

Express the function $f(t)$ as a Fourier series and write down the first four non-zero terms of the series.

This page is for rough work.

Formula sheet

Table 1. Common functions and their Laplace transform

Function	Laplace transform	Function	Laplace transform
1	$\frac{1}{s}$	$e^{-at} \sin bt$	$\frac{b}{(s+a)^2 + b^2}$
t^n	$\frac{n!}{s^{n+1}}$	$e^{-at} \cos bt$	$\frac{s+a}{(s+a)^2 + b^2}$
e^{at}	$\frac{1}{s-a}$	$t \sin bt$	$\frac{2bs}{(s^2 + b^2)^2}$
e^{-at}	$\frac{1}{s+a}$	$t \cos bt$	$\frac{s^2 - b^2}{(s^2 + b^2)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$\sinh t$	$\frac{b}{s^2 - b^2}$
$\sin bt$	$\frac{b}{s^2 + b^2}$	$\cosh t$	$\frac{s}{s^2 - b^2}$
$\cos bt$	$\frac{s}{s^2 + b^2}$		

Table 2. The Laplace transform of derivatives

$\mathcal{L}\{f'(t)\}$	$sF(s) - f(0)$
$\mathcal{L}\{f''(t)\}$	$s^2F(s) - sf(0) - f'(0)$
$\mathcal{L}\{f'''(t)\}$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Newton's Interpolation	$y_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \dots + \frac{p(p-1)\dots(p-n+1)}{n!}\Delta^n y_0$ $y_n(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \dots + \frac{p(p+1)\dots(p+n-1)}{n!}\nabla^n y_n$
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Fourier series	$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T} \right)$
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MLO and Bloom's Level of Complexity

Q #	MLO Addressed	Complexity Level	Mark	Remark
1	2, 3	Application, Analysis	8	
2	1	Application	6	
3	3, 4	Application, Analysis	8	
4	2,3	Application, Analysis	5	
5	1	Application	6	
6	1	Application	7	

References:

1. Croft, T. and Davison, R. (2020). Mathematics for Engineers (Fifth Edition). UK: Pearson.
2. Loughborough University Mathematics Learning Support Centre. (n.d.). *HELM Workbooks*. Retrieved October 12, 2025, <https://www.lboro.ac.uk/departments/mlsc/student-resources/helm-workbooks/>