الـجـامـعـة الوطـنيـة
National University
Science & Technology للعلـــوم والتكنـــولوجيا
كليـــة عمـــان البحـــرية الـــدولية
International Maritime College Oman

Final Exam
PT-TMATH-III: APPLIED MATHS FOR PROCESS ENGINEERING
Fall 2024

Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration: 2.5 hours
Instructor's Name: Dr. Taofeek Olanrewaju Alade
Exam Date: 05/01/2025
Program: PE

	40
	10

Student Information			
Name:		ID:	
Signature:			

Invigilator		
Initials:		☐ Student ID checked
Time received:		

Question 1 **[8 marks]**

In a process optimization study for a city's development, the population of a town over the past six censuses is recorded as follows:

Year (*x*)	1971	1981	1991	2001	2011	2021
Population (y) (in thousands)	12	15	20	27	39	52

Using this data, estimate the population for the year 2006 using interpolation method.
Write your answer to the nearest whole number.

Question 2 **[8 marks]**

In a periodic process system, the function $f(t)$ is defined as:

$$f(t) = \begin{cases} -4, & -\pi < t \leq 0 \\ 4, & 0 < t < \pi \end{cases}$$

and $f(t + 2\pi) = f(t)$ indicating that the function is periodic with a period of 2π.

Analyze the periodic behavior of the system by determining the Fourier series representation of $f(t)$.

Question 3 **[8 marks]**

The matrix A is given by

$$A = \begin{pmatrix} 1 & -1 & 2 \\ -3 & -2 & 3 \\ 2 & -1 & 1 \end{pmatrix}$$

(i) Determine the determinant of A or |A|. (2 marks)

(ii) Find the characteristic equation and the eigenvalues of A. (4 marks)

(iii) From (ii), Calculate any one eigenvector of matrix A. (2 marks)

Question 4 **[6 marks]**

In a chemical plant, a tank is used to mix chemicals. Due to a sudden change in the flow of chemicals into the tank, the plant engineer needs to figure out how the concentration of the chemicals will change over time.

The change in concentration is described by the formula:

$$\frac{-4s + 3^2}{6s^2 + 24s + 120}$$

Can you help the engineer predicts how the concentration changes by finding the inverse Laplace transform of this formula?

Question 5 **[4 marks]**

Determine the Laplace transform of the following functions

(i) $\frac{1}{3}\cos 6t - 2e^{t} + \sin t$ (2 marks)

(ii) $6 - 3\cos\frac{2t}{3} + 2\cos t$ (2 marks)

Question 6 **[6 marks]**

In a chemical process involving gas flow, the pressure p of a gas change with altitude y according to the following equation:

$$\frac{dp}{dy} = -k\ p$$

where k is a constant, and the pressure at ground level $(y = 0)$ is given as p_0 .

Solve this equation using the Laplace transform method to determine the pressure p as a function of altitude y.

Table of Laplace Transform

Function, f(t)	Laplace Transform $\{f(t)\} = F(s)$	Function, f(t)	Laplace Transform $\{f(t)\} = F(s)$
t^n	$\frac{n!}{s^{n+1}}$	$sinh\ bt$	$\frac{b}{s^2 - b^2}$
$e^{a\,t}$	$\frac{1}{s - a}$	$cosh\ bt$	$\frac{s}{s^2 - b^2}$
$t^n e^{-\,a\,t}$	$\frac{n!}{(s+a)^{n+1}}$	$e^{-\,a\,t}\ sinh\ bt$	$\frac{b}{(s+a)^2 - b^2}$
$sin\ bt$	$\frac{b}{s^2+b^2}$	$e^{-\,a\,t}\ cosh\ bt$	$\frac{s+a}{(s+a)^2 - b^2}$
$cos\ bt$	$\frac{s}{s^2+b^2}$	$t\ sin\ bt$	$\frac{2bs}{(s^2+b^2)^2}$
$e^{-\,a\,t}\ sin\ bt$	$\frac{b}{(s+a)^2+b^2}$	$t\ cos\ bt$	$\frac{s^2 - b^2}{(s^2+b^2)^2}$
$e^{-\,a\,t}\ cos\ bt$	$\frac{s+a}{(s+a)^2+b^2}$		

Laplace Transform

$\{y'(t)\}$	$s\,Y(s) - y(0)$
$\{y''(t)\}$	$s^2\,Y(s) - s\,y(0) - y'(0)$

Newton's backward interpolation formula

$$y_n(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n \ldots\ldots + \frac{p(p+1)\cdots\cdots(p+n-1)}{n!}\nabla^n y_n$$

Newton's forward interpolation formula

$$y_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 \ldots\ldots + \frac{p(p-1)\cdots\cdots(p-n+1)}{n!}\Delta^n y_0$$

Fourier series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n cos\, nt + b_n sin\, nt)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos nt \, dt$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin nt \, dt$$

MLO & Bloom's Level of Complexity

Q #	MLO Addressed	Complexity Level	Mark	Remark
1, 4	**2, 3, 4, 5**	Application	**14**	
3	**1,**	Understanding/ Analysing	**8**	
2	**4**	Evaluating	**8**	
6	**2**	Analysing	**6**	
5	**1, 3**	Remembering	**4**	