الجامعة الوطنية
National University
Science & Technology للعلوم والتكنولوجيا
كلية عمان البحرية الدولية
International Maritime College Oman

ACADEMIC YEAR 2023 - 2024

Program	Year	Semester	Paper
PE	2023	1	MAIN

MODULE NAME:	**Technical Math I**		
MODULE CODE:	**TMATH-1**	**EXAM DATE:**	**24.12.2023**
INSTRUCTOR's NAME:	**Dr. Amer Alhabsi**	**DURATION:**	**2.5 hrs.**

Questions to be answered on:	**Allowed tools:**	**Number of pages**
✓ Space provided on the question paper	Pen, Pencil & Calculator	(Incl. cover page): **9**

Points of attention:

- For each question, the maximum earned points are mentioned between brackets at the end of each question.
- Write very clearly! Answers that are not readable are not marked and don't get points!
- Make sure your answers are written to the point.
- All answers should be written **in English.**
- Write all the answers in **blue or black pen only.**
- Use the **pencil** only for **diagrams & graphs.**
- Show all the calculation steps in the given space.
- When finished submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

STUDENT NAME:

STUDENT ID:

FINAL MARKS

	50

Number of answer scripts:……..……. **Invigilator:**……..……..…

Student's signature: …… **Time of receipt:**……..…

[15 pts]

1. Answer the following:

   (a) Calculate $\lim_{x \to 2} \dfrac{x^3 - 2x^2 + x - 2}{x^2 + 4x - 12}$

   (b) given that,

   $$x^2 y^5 + xy = 8,$$

   find $\dfrac{dy}{dx}$

(c) Find the equation o the tangent to the curve $y = x^3 - 2x + 1$ at $x = 2$.

[10 pts]

2. Air is pumped into a spherical balloon at a rate of $2\,\text{cm}^3/\text{s}$.

   (a) What is the rate of change of the radius when the radius is $20\,\text{cm}$

   (b) What is the rate of change of the surface area at the same moment as that of part (a).

[8 pts]

3. Calculate the area bounded by the x-axis and the curve

$$y = x^2 \ln x$$

from $x = 1$ to $x = 10$. Write your answer to three decimal places.

[7 pts]

4. The volume of gas V and its pressure P are related by

$$P = 1000V^{-1.5}.$$

The work, W, done when the gas expands from volumes v_1 to v_2 is

$$W = \int_{v_1}^{v_2} P\,dV$$

Calculate the work done when the gas expands from 10 to 20 units.

[10 pts]

5. **Answer ONE of the following questions**

1. Newton's law of cooling states that the temperature of an object $T(t)$ changes as per the differential equation

$$\frac{dT}{dt} = -k(T - A)$$

where k is a constant, A is the ambient temperature and t is the time in seconds. Let an object with temperature 373 K be placed in a room with temperature 293 K .

(a) Solve the differential equation

(b) what will be the temperature of the body after 200 seconds

(c) what will be the temperature of the body after a very long time.

2. For the following differential equation:

$$x^3\frac{dy}{dx} + x^2y = x$$

(a) determine the integrating factor $\mu(x)$.

(b) solve the differential equation.

(c) determine the constant of integration given the condition $y(1) = 3$.

Table of Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C,\ \ n \neq -1 \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int a^x\,dx = \frac{1}{\ln a}a^x + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \tan x\,dx = \ln|\sec x| + C \qquad \int \sec^2 x\,dx = \tan x + C$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$$

Q #	MLO Addressed	Complexity Level	Mark	Comments
1	1	3	15	
2	1, 2	4	10	
3	1, 4	3	8	
4	1, 3, 4	4	7	
5	1, 3, 4, 6	3	10	

Volume of a sphere with radius r is

$$V = \frac{4}{3}\pi r^3$$

Surface area of a sphere with radius r is

$$A = 4\pi r^2$$