

Final Exam
TMATH-II: TECHNICAL MATH II
Spring 2025

Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration: 2.5 hours
Instructor's Name: Dr. Rokhsaneh Yousef Zehi
Exam Date: 18/06/2025
Program: PE

	40
	10

Student Information

Name: ID:
Signature:

Invigilator

Initials: ☐ Student ID checked
Time received:

Question 1**[7 marks]**

In a gas storage tank, pressure is regulated using a venting valve. The pressure $P(t)$ inside the tank (in kPa) decreases over time as gas escapes. The rate of pressure drop is proportional to the difference between the current pressure and the ambient pressure. This behavior is modeled by the differential equation:

$$\frac{dP}{dt} = -k(P - 100)$$

where k is a positive constant.

- Find the general solution for $P(t)$. (3 marks)
- Given that the initial pressure is 300 kPa and it drops to 200 kPa in 10 minutes, find a particular solution for $P(t)$. (3 marks)
- Determine the pressure in the tank after 30 minutes. (1 mark)

Question 2**[8 marks]**

A mechanical system consists of a mass attached to a spring and a damping element. The displacement $x(t)$ of the mass from its equilibrium position is modeled by the following second-order differential equation:

$$\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x = t + 4e^{3t} + 2$$

Obtain the particular solution to this differential equation that satisfies the initial conditions $x(0) = 3, x'(0) = 1$.

Question 3**[6 marks]**

Determine the position and nature of the stationary point of the following function.

$$f(x, y) = x^3 + y^2 - 3x - 3y + 3xy$$

Question 4**[6 marks]**

The concentration of a chemical substance in a reactor at any point is modeled by the equation:

$$C(x, y) = \frac{y^2 + e^{xy}}{(x + 2y)^2}$$

Where x represent the distance along the reactor (in meters), and y is the radial distance from the center (in meters).

- a. Find the partial derivatives $\partial C / \partial x$ and $\partial C / \partial y$. (5 marks)
- b. Evaluate both partial derivatives at the point $(x, y) = (2, 2)$. (1 mark)

Question 5**[7 marks]**

In a system of three tanks, the flow rates f_1 , f_2 and f_3 (in liters per minute) are related by the following equations, which represent the inflow and outflow rates between the tanks:

$$f_1 + 2f_2 - f_3 = 8$$

$$f_1 + f_2 + 2f_3 = 10$$

$$2f_1 - f_3 = 9$$

Using Cramer's rule, evaluate the three flow rates.

Question 6**[6 marks]**

Determine the inverse of the following matrix.

$$A = \begin{pmatrix} 5 & 2 & -1 \\ 6 & 3 & 1 \\ 5 & 9 & 6 \end{pmatrix}$$

MLO and Bloom's Level of Complexity

Q #	MLO Addressed	Complexity Level	Mark	Remark
1	3,4	Application, Analysis	7	
2	3,4	Application, Analysis	8	
3	1	Application	6	
4	2,3	Application, Analysis	6	
5	1,2	Application	7	
6	1	Application, Analysis	6	

References:

1. Anthony Croft, R. D., 2015. Mathematics for Engineers. Fourth ed. Harlow: Pearson.
2. J. Washington, A., 2014. Basic Technical Mathematics with Calculus. 10 ed. Harlow: Pearson Education Limited.
3. Stewart, J., 2008. *Calculus: Early Transcendentals*. 6th ed. Boston: Brooks/Cole.