

Final Exam
TMATH-III: APPLIED MATHS FOR PROCESS ENGINEERING
Fall 2025

Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration: 3 hours
Instructor's Name: Dr. Rokhsaneh Yousef Zehi
Exam Date: 25/12 /2025
Program: PE

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Student Information	
Name:	<input type="text"/>
Signature:	<input type="text"/>
ID:	<input type="text"/>

Invigilator	
Initials:	<input type="text"/>
Time received:	<input type="text"/>
<input type="checkbox"/> Student ID checked	

Question 1**[6 marks]**

A thermal system consists of three interconnected metal plates whose temperature deviations from ambient (in °C) are denoted by $T_1(t)$, $T_2(t)$, $T_3(t)$. The heat exchange between the plates is described by the following system of differential equations:

$$\frac{dT_1}{dt} = -3T_1 + T_2$$

$$\frac{dT_2}{dt} = 2T_1 - 4T_2$$

$$\frac{dT_3}{dt} = T_1 + T_2 - 3T_3$$

where t is time in minutes.

(a) Write the system in matrix form $T' = AT$ and determine the eigenvalues of the matrix A .

(4 marks)

(b) Determine **only one** of the eigenvectors of A .

(2 marks)

Question 2**[6 marks]**

(a) Determine the Laplace transform of the following function:

(2 mark)

$$\frac{3e^{5t} \sin 4t}{2e^{3t}}$$

(b) Determine the inverse Laplace transform of the following function:

(4 marks)

$$\frac{-(s + 7)}{2s^2 + 8s + 10}$$

Question 3**[7 marks]**

A constant voltage of 18 V is applied to a series RL circuit containing a 3 H inductor and a $6\ \Omega$ resistor. In addition to the constant supply, a decaying transient voltage source from another part of the system contributes an extra term of $6e^{-3t}$ volts.

The resulting differential equation describing the current $i(t)$ in the circuit is:

$$3 \frac{di}{dt} + 6i = 18 + 6e^{-3t}$$

The circuit starts from rest; $i(0) = 0$.

Using the Laplace transform method, find $i(t)$ and then determine the value of the current at $t = 5$ seconds.

Question 4**[6 marks]**

In a flow system, the pressure drop ΔP (*kpa*) across a long, narrow pipe is directly proportional to the volumetric flow rate Q (*L/s*). The relationship is assumed to be:

$$\Delta P = aQ + b$$

The following experimental data were recorded:

Flow rate Q (<i>L/s</i>)	2	4	6	8	10	12	14	16
Pressure drop ΔP (<i>kpa</i>)	18	28	35	47	58	66	78	86

- (a) Using the least squares method, determine the constants a and b . (5 marks)
- (b) Estimate the pressure drop when the flow rate is $Q = 5$ *L/s*. (0.5 marks)
- (c) Determine the flow rate when the pressure drop is $\Delta P = 45$ *kpa*. (0.5 marks)

Question 5**[4 marks]**

For each of the following sequences $x[k]$, find the limit to infinity (if it exists). Then state whether the sequence converges or diverges.

(a) $x[k] = \frac{\ln(k^2+1)}{3k}$ (2 marks)

(b) $x[k] = \frac{3k^2+5k}{k-2k^3}$ (2 marks)

Question 6**[5 marks]**

Determine the first 4 non-zero terms of Maclaurin series expansion for the following function:

$$f(x) = \frac{-\cos 2x}{x^2}$$

Question 7**[6 marks]**

Let

$$f(t) = \begin{cases} 0 & -2 < t < 0 \\ 3 & 0 < t < 2 \end{cases}$$

and suppose that $f(t)$ is periodic with period $T = 4$, so that $f(t + 4) = f(t)$. Express the function $f(t)$ as a Fourier series and write down the first four non-zero terms of the series.

This page is rough work.

Formula sheet

Table1. Common functions and their Laplace transform

Function	Laplace transform	Function	Laplace transform
1	$\frac{1}{s}$	$e^{-at} \sin bt$	$\frac{b}{(s+a)^2 + b^2}$
t^n	$\frac{n!}{s^{n+1}}$	$e^{-at} \cos bt$	$\frac{s+a}{(s+a)^2 + b^2}$
e^{at}	$\frac{1}{s-a}$	$t \sin bt$	$\frac{2bs}{(s^2 + b^2)^2}$
e^{-at}	$\frac{1}{s+a}$	$t \cos bt$	$\frac{s^2 - b^2}{(s^2 + b^2)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$\sinh t$	$\frac{b}{s^2 - b^2}$
$\sin bt$	$\frac{b}{s^2 + b^2}$	$\cosh t$	$\frac{s}{s^2 - b^2}$
$\cos bt$	$\frac{s}{s^2 + b^2}$		

Table 2. The Laplace transform of derivatives

$\mathcal{L}\{f'(t)\}$	$sF(s) - f(0)$
$\mathcal{L}\{f''(t)\}$	$s^2F(s) - sf(0) - f'(0)$
$\mathcal{L}\{f'''(t)\}$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Normal Equations		
	$a \sum x^2 + b \sum x = \sum xy$	$a \sum x^4 + b \sum x^3 + c \sum x^2 = \sum x^2y$
	$a \sum x + bn = \sum y$	$a \sum x^3 + b \sum x^2 + c \sum x = \sum xy$
		$a \sum x^2 + b \sum x + cn = \sum y$

Fourier series	$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T} \right)$
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MLO and Bloom's Level of Complexity

Q #	MLO Addressed	Complexity Level	Mark	Remark
1	2, 3	Application, Analysis	7	
2	1	Application	6	
3	3, 4	Application, Analysis	7	
4	2,3	Application, Analysis	5	
5	1	Application, Analysis	4	
6	1	Application	5	
7	1	Application	6	

References:

1. Croft, T. and Davison, R. (2020). Mathematics for Engineers (Fifth Edition). UK: Pearson.
2. Loughborough University Mathematics Learning Support Centre. (n.d.). *HELM Workbooks*. Retrieved October 12, 2025, <https://www.lboro.ac.uk/departments/mlsc/student-resources/helm-workbooks/>