الـجـامـعـة الوطنيـة
National University
Science & Technology للعلـوم والتكنـولوجيا

كليـة عمـان البحـرية الـدولية
International Maritime College Oman

Final Exam

TMATH-III: APPLIED MATHS FOR PROCESS ENGINEERING

Fall 2025

Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration: 3 hours
Instructor's Name: Dr. Rokhsaneh Yousef Zehi
Exam Date: 25/12 /2025
Program: PE

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Student Information			
Name:		ID:	
Signature:			

Invigilator		
Initials:		☐ Student ID checked
Time received:		

Question 1 **[6 marks]**

Three connected tanks contain saline solutions. The amount of salt (in kg) in tank 1, tank 2, and tank 3 at time t minutes are denoted by $y_1(t), y_2(t), y_3(t)$ respectively. The mixing and flow process leads to the following system of differential equations:

$$\frac{dy_1}{dt} = -4y_1 + 2y_2$$
$$\frac{dy_2}{dt} = 3y_1 - 5y_2$$
$$\frac{dy_3}{dt} = y_1 + y_2 - 3y_3$$

(a) Write the system in matrix form $\mathrm{Y}' = A\mathrm{Y}$ and determine the eigenvalues of the matrix A.

(4 marks)

(b) Determine **only one** of the eigenvectors of A. (2 marks)

Question 2 **[6 marks]**

(a) Determine the Laplace transform of the following function: (2 mark)

$$\frac{5e^{4t}cos3t}{3e^{-2t}}$$

(b) Determine the inverse Laplace transform of the following function: (4 marks)

$$\frac{3s+12}{2s^2+8s+16}$$

Question 3 **[7 marks]**

A constant voltage of $20\,V$ is applied to an RC circuit in which the voltage across the capacitor is denoted by $v(t)$. The resistor-capacitor combination produces the following differential equation:

$$\frac{dv}{dt} + 4v = 20 + 10e^{-2t}$$

The circuit starts from rest; $v(0) = 0$.

Using the Laplace transform method, find $v(t)$ and then determine the value of the voltage at t = 10 seconds.

Question 4 **[6 marks]**

Let's consider a heat exchanger where we want to model how the outlet temperature (in ℃) of a fluid changes with flow rate (kg/s). We ran the exchanger at several flow rates $Q({}^{kg}/_{s})$ and recorded outlet temperature T_{out} (℃).

Flow rate	2	4	6	8	10	12	14
Outlet Temperature	100	96	92	88	85	82	79

Over this operating window the relation looks roughly linear, so fits:

$$T_{out} = aQ + b$$

(a) Using the least square method, determine the constants a and b. (5 marks)

(b) Estimate the outlet temperature when the flow rate is $11\ kg/s$. (0.5 marks)

(c) Determine the flow rate when the outlet temperature is 90℃. (0.5 marks)

Question 5 **[4 marks]**

For each of the following sequences $x[k]$, find the limit to infinity (if it exists). Then state whether the sequence converges or diverges.

(a) $\boldsymbol{x[k] = \frac{\ln\left(\frac{1}{k}\right)}{2k}}$ (2 marks)

(b) $\boldsymbol{x[k] = \frac{5\mathrm{k}-3k^2-3}{2k^4-4k}}$ (2 marks)

Question 6 **[5 marks]**

Determine the first 4 non-zero terms of Taylor series expansion for $f(x) = (x - \frac{\pi}{6})^2 \cos 2x$ about the point $x = \frac{\pi}{6}$.

Question 7 **[6 marks]**

Let

$$f(t) = \begin{cases} 0 & -3 < t < 0 \\ 2 & 0 < t < 3 \end{cases}$$

and suppose that $f(t)$ is periodic with period $T = 6$, so that $f(t+6) = f(t)$. Express the function $f(t)$ as a Fourier series and write down the first four non-zero terms of the series.

This page is for rough work.

Formula sheet

Table1. Common functions and their Laplace transform

Function	Laplace transform	Function	Laplace transform
$\mathbf{1}$	$\frac{1}{s}$	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
t^n	$\frac{n!}{s^{n+1}}$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
e^{at}	$\frac{1}{s-a}$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
e^{-at}	$\frac{1}{s+a}$	$t\,sos\,bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$\sinh t$	$\frac{b}{s^2-b^2}$
$\sin bt$	$\frac{b}{s^2+b^2}$	$\cosh t$	$\frac{s}{s^2-b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$		

Table 2. The Laplace transform of derivatives

$\mathcal{L}\{f'(t)\}$	$sF(s)-f(0)$
$\mathcal{L}\{f''(t)\}$	$s^2F(s)-sf(0)-f'(0)$
$\mathcal{L}\{f'''(t)\}$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

Normal Equations	$a\sum x^2+b\sum x=\sum xy$ $a\sum x+bn=\sum y$	$a\sum x^4+b\sum x^3+c\sum x^2=\sum x^2y$ $a\sum x^3+b\sum x^2+c\sum x=\sum xy$ $a\sum x^2+b\sum x+cn=\sum y$

Fourier series	$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2n\pi t}{T} + b_n \sin\frac{2n\pi t}{T}\right)$$

MLO and Bloom's Level of Complexity

Q #	MLO Addressed	Complexity Level	Mark	Remark
1	**2, 3**	Application, Analysis	**6**	
2	**1**	Application	**6**	
3	**3, 4**	Application, Analysis	**7**	
4	**2,3**	Application, Analysis	**6**	
5	**1**	Application, Analysis	**4**	
6	**1**	Application	**5**	
7	**1**	Application	**6**	

References:

1. Croft, T. and Davison, R. (2020). Mathematics for Engineers (Fifth Edition). UK: Pearson.
2. Loughborough University Mathematics Learning Support Centre. (n.d.). *HELM Workbooks*. Retrieved October 12, 2025, https://www.lboro.ac.uk/departments/mlsc/student-resources/helm-workbooks/