الجامعة الوطنية
National University
Science & Technology للعلوم والتكنولوجيا
كلية عمان البحرية الدولية
International Maritime College Oman

ACADEMIC YEAR 2023 - 2024

Program	Year	Semester	Paper
PE	3	1	MAIN

MODULE NAME:	**Applied Maths for Process Engineering**		
MODULE CODE:	**TMATH-III**	**EXAM DATE:**	**31/12/2023**
INSTRUCTOR's NAME:	**Dr. Taofeek**	**DURATION:**	**2 hrs.**

Questions to be answered on:	**Allowed tools:**	**Number of pages**
✓ Space provided on the question paper	Pen, Pencil & Calculator	(Incl. cover page): **11**

Points of attention:

- For each question, the maximum earned points are mentioned between brackets at the end of each question.
- Write very clearly! Answers that are not readable are not marked and don't get points!
- Make sure your answers are written to the point.
- All answers should be written **in English.**
- Write all the answers in **blue or black pen only.**
- Use the **pencil** only for **diagrams & graphs.**
- Show all the calculation steps in the given space.
- When finished submit the question paper, together with the answer scripts and the signed cover page to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

STUDENT NAME:

STUDENT ID:

FINAL MARKS

	40
	10

Number of answer scripts:……..…….

Invigilator:……..……..

Student's signature: ……

Time of receipt:……..…..

ANSWER ALL THE QUESTIONS

Question 1 **[6 marks]**

The matrix A is given by

$$A = \begin{pmatrix} 4 & -1 & 1 \\ -2 & 4 & 0 \\ -4 & 3 & 1 \end{pmatrix}$$

(i) Determine the characteristic equation and the eigenvalues of A. [4 marks]
(ii) From (i), Calculate any one eigenvector of matrix A. [2 marks]

Question 2 **[4 marks]**

Determine the Laplace transform of the following functions

(i) $\frac{1}{3}cos\ 6t - 2e^{-t}\ sin\ t$ [2 marks]

(ii) $2 - 3\ cos\frac{2t}{3} + 2t\ cos\ 2t$ [2 marks]

Question 3 **[6 marks]**

Find the inverse Laplace transform of the following

$$\frac{2s-1}{s^2+6s+10}$$

Question 4 **[6 marks]**

The pressure p, of a gas varies with altitude, x according to the equation

$$\frac{dp}{dx} = -k\,p$$

Where k is constant, the pressure at ground level ($x = 0$) is known to be p_0.
Solve the equation using Laplace transform method to find p in terms of x.

Question 5 **[10 marks]**

Let us consider a mass-spring oscillation system equation

$$m\,x'' + c\,x' + k\,x = f(t)$$

Given that $m = 1$, $c = 0$ and $k = 3$, and

$$f(t) = \begin{cases} 0, & -1 < t < 0 \\ 2, & 0 < t < 1 \end{cases}$$

$$f(t+2) = f(t)$$

Determine the solution of the equation using Fourier series.

Question 6 **[8 marks]**

The population of a town in the last six censuses was as given below. Estimate the population for the year 1946 to the nearest whole number.

Year(x)	1911	1921	1931	1941	1951	1961
Population in thousands(y)	12	15	20	27	39	52

-

Table of Laplace Transform

Function, f(t)	Laplace Transform $\{f(t)\} = F(s)$	Function, f(t)	Laplace Transform $\{f(t)\} = F(s)$
t^n	$\frac{n!}{s^{n+1}}$	$sinh\ bt$	$\frac{b}{s^2 - b^2}$
$e^{a\,t}$	$\frac{1}{s - a}$	$cosh\ bt$	$\frac{s}{s^2 - b^2}$
$t^n e^{-a\,t}$	$\frac{n!}{(s+a)^{n+1}}$	$e^{-a\,t}\ sinh\ bt$	$\frac{b}{(s+a)^2 - b^2}$
$sin\ bt$	$\frac{b}{s^2 + b^2}$	$e^{-a\,t}\ cosh\ bt$	$\frac{s+a}{(s+a)^2 - b^2}$
$cos\ bt$	$\frac{s}{s^2 + b^2}$	$t\ sin\ bt$	$\frac{2bs}{(s^2 + b^2)^2}$
$e^{-a\,t}\ sin\ bt$	$\frac{b}{(s+a)^2 + b^2}$	$t\ cos\ bt$	$\frac{s^2 - b^2}{(s^2 + b^2)^2}$
$e^{-a\,t}\ cos\ bt$	$\frac{s+a}{(s+a)^2 + b^2}$		

Laplace Transform

$\{f'(t)\}$	$\boldsymbol{s}\,F(\boldsymbol{s}) - f(0)$
$\{f''(t)\}$	$\boldsymbol{s^2}\,F(\boldsymbol{s}) - \boldsymbol{s}\,f(0) - f'(0)$
$\{f'''(t)\}$	$s^3\,F(\boldsymbol{s}) - s^2\,f(0) - s\,f'(0) - f''(0)$

Newton's backward interpolation formula

$$y_n(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n \cdots\cdots + \frac{p(p+1)\cdots\cdots(p+n-1)}{n!}\nabla^n y_n$$

Newton's forward interpolation formula

$$y_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 \cdots\cdots + \frac{p(p-1)\cdots\cdots(p-n+1)}{n!}\Delta^n y_0$$

MLO & Bloom's Level of Complexity

Q #	MLO Addressed	Complexity Level	Mark	Remark
4,5	**2, 3, 4, 5**	Application	**12**	
6	**1,**	Understanding/ Analysing	**8**	
2, 3	**4**	Evaluating	**10**	
5	**2**	Analysing	**4**	
1	**1, 3**	Remembering	**6**	
XX				
XX				
XX				
XX				