

FINAL EXAM
FMTH005: PURE MATHEMATICS
Fall-T2 2024 – 25

Points of attention:

- For each question, the maximum earned points are specified in the question.
- Write clearly! Answers that are not readable are not marked and don't earn marks!
- All answers should be written in English using **blue or black pens** only.
- Use the pencil only for diagrams and graphs.
- Show all the calculation steps in the given space.
- When finished, submit and sign the question paper to the invigilator.
- Any cheating/copying may result in an instant failing of the examination.

Exam Duration: 2 hours
Instructor's Name: Muhammad Kazam
Exam Date: 09/02/2025
Program: FND

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Student Information

Name: ID:
Signature:

Invigilator

Initials: ☐ Student ID checked
Time received:

Question 1.**[10 Marks]****Circle the correct option to fill in the blanks.****Example.** 85 is a _____ digit number.

- a. 1 **b. 2** c. 3 d. 4

1. The domain of the function $f(x) = \frac{1}{\sqrt{3-x}}$ is _____.

- a. $\mathbb{R}$ b. $\{x|x \leq 3\}$ c. $\{x|x < 3\}$ d. $\{x|x > 3\}$

2. $\frac{\sin^2 \theta}{\cos^2 \theta} =$ _____

- a. $\cot^2 \theta$ b. $\sec^2 \theta$ c. $\sec^2 \theta - 1$ d. $1 + \sec^2 \theta$

3. If $\sin \theta = \frac{2}{3}$, then $\cot \theta =$ _____.

- a. $\frac{3}{\sqrt{5}}$ b. $\frac{2}{\sqrt{5}}$ c. $\frac{\sqrt{5}}{3}$ d. $\frac{\sqrt{5}}{2}$

4. If an event cannot happen, its probability will be _____.

- a. 0 b. $\frac{1}{2}$ c. $\frac{1}{3}$ d. 1

5. The mode of the data 60, 74, 60, 62, 68, 68, 79, 62, 59, 68, 75, 62 is _____.

- a. 60 and 68 b. 62 and 68 c. 60 and 62 d. 60, 62 and 68

6. The class mark for the group (12 – 19) is _____.

- a. 11.5 – 19.5 b. 12 c. 19 d. 15.5

7. The axis of symmetry of the curve $f(x) = x^2 + x + 2$ is at $x =$ _____.

- a. 1 b. $\frac{1}{2}$ c. $-\frac{1}{2}$ d. -1

8. If $g(x) = \frac{1-x}{1+x}$, then $g(-a) =$ _____.

- a. $\frac{a-1}{a+1}$ b. $\frac{a+1}{a-1}$ c. $\frac{a+1}{-a+1}$ d. $\frac{-a-1}{1-a}$

9. $a^{\log_a x} =$ _____

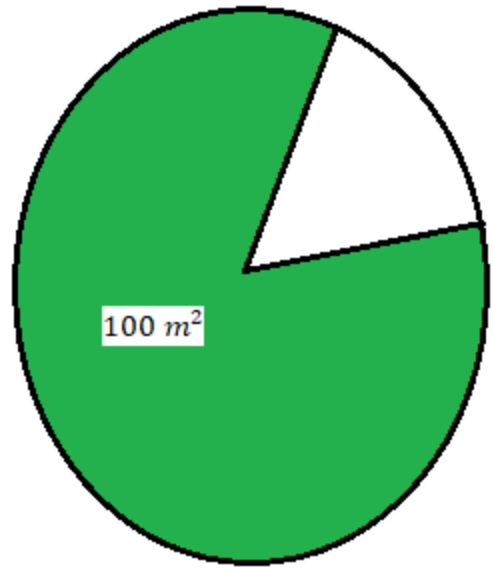
- a. 0 b. a c. x d. $\log_a x$

10. If $5^{x+1} = 1$, then $x =$ _____.

- a. 0 b. 1 c. -1 d. 2

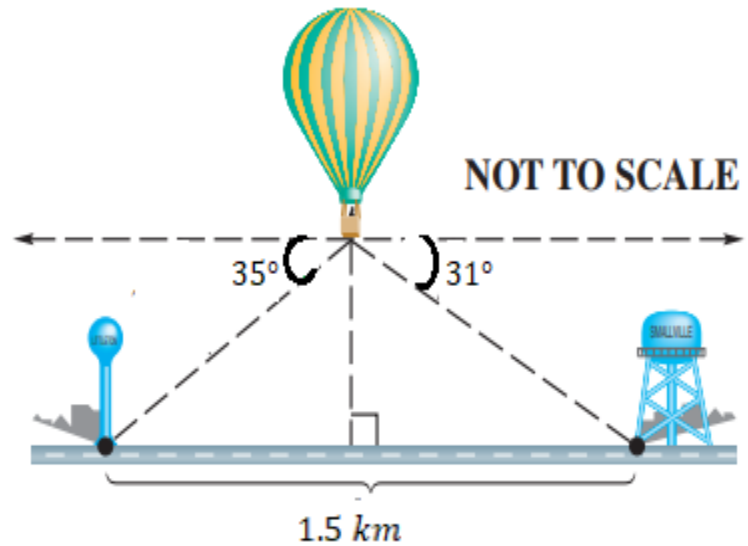
Question 2.**[4 Marks]**

An engineer is asked to design a water sprinkler that will cover a field of 100 m^2 that is in the shape of a sector of a circle of radius 6.5 m . Calculate the angle the sprinkler rotate in degrees.



Question 3.**[5 Marks]**

A balloonist is directly above a straight road 1.5 km long that joins two villages. She finds that the town closer to her is at an angle of depression of 35° , and the farther town is at an angle of depression of 31° . Calculate the height of the balloon above the ground.



(Lial, Schneider and Daniels, 2017)

Question 4.**[4 Marks]**

Prove the trigonometric identities given below.

$$(\sec \theta + \csc \theta)(\cos \theta - \sin \theta) = \cot \theta - \tan \theta$$

Question 5.**[3 Marks]**

Use the Laws of Logarithms to combine the expression given below. Simplify them the lowest of the terms.

$$\frac{1}{3}\log x^4y^5 - \frac{3}{4}\log x^2y$$

Question 6.**[6 Marks]**

Solve the equations given below for the variable x .

a. $\log_7(3x + 2) - \log_7(x - 2) = 1$

(2 marks)

b. $T = T_0 + (T_1 - T_0)10^{-kx}$

(4 marks)

Question 7.**[4 Marks]**

The total amount spent by Americans on shoes and clothing from 2000 to 2013 can be modeled by $f(x) = 0.7714x^2 - 3.693x + 297.9$, where $x = 0$ represents 2000 and $f(x)$ is in billions of dollars.

a. In what year did spending on shoes and clothing reach a minimum?

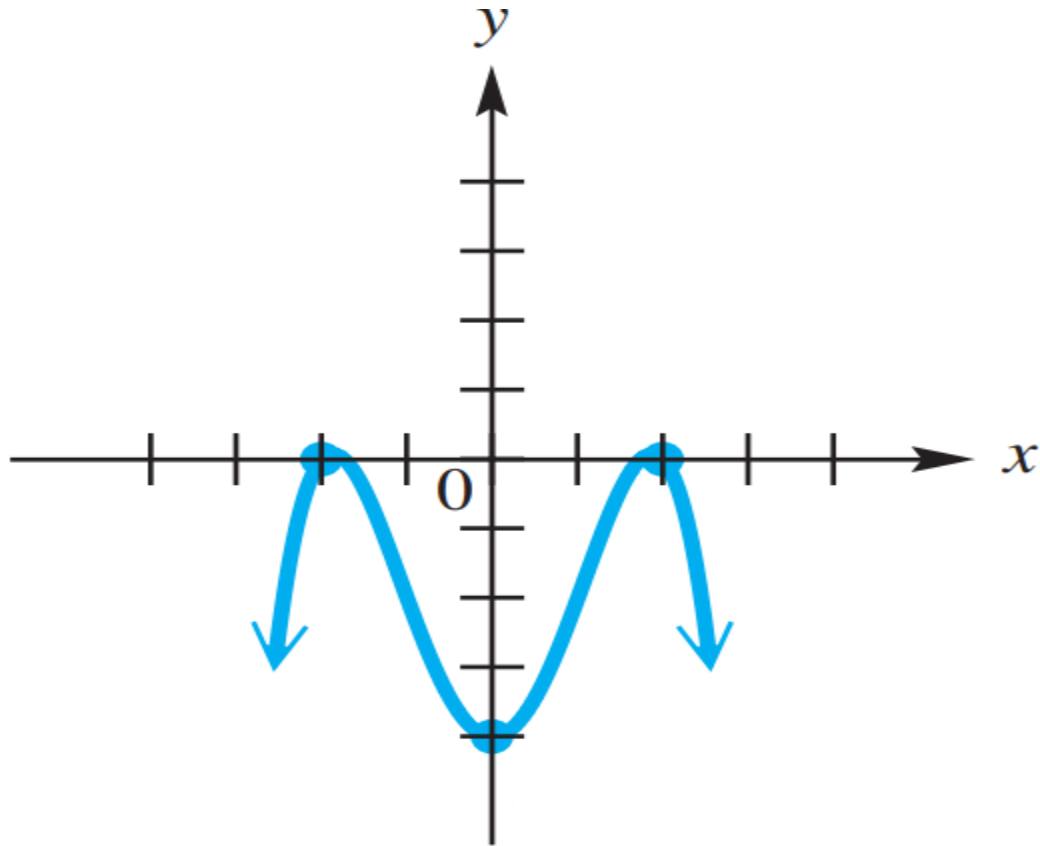
(2 marks)

b. Determine the minimum amount spent on shoes and clothing.

(2 marks)

Question 8.**[4 Marks]**

The graph of a function f is given below.



- a. Use the graph to find all the local maximum and minimum values of the function and the value of x at which each occurs. **(2 marks)**
- b. Use the graph to find the intervals on which the function is increasing and on which the function is decreasing. **(2 marks)**

Formula Sheet

Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Law of Sines	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Law of Cosines	$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos A \\b^2 &= a^2 + c^2 - 2ac \cos B \\c^2 &= a^2 + b^2 - 2ab \cos C\end{aligned}$

Reference:

Lial, M.L., Schneider, D.I. and Daniels, C.J. (2017) *College Algebra & Trigonometry*. 6th edn. Harlow: Pearson.